

**Part 1: Multiple Choice.** Circle the letter corresponding to the best answer.

- In a test of  $H_0: p = 0.7$  against  $H_a: p \neq 0.7$ , a sample of size 80 produces  $z = 0.8$  for the value of the test statistic. Which of the following is closest to the  $P$ -value of the test?
  - 0.2090
  - 0.2119
  - 0.4238
  - 0.4681
  - 0.7881
- An opinion poll asks a simple random sample of 100 college seniors how they view their job prospects. In all, 53 say "good." Does the poll give convincing evidence to conclude that more than half of all seniors think their job prospects are good? If  $p$  = the proportion of all college seniors who say their job prospects are good, what are the hypotheses for a test to answer this question?
  - $H_0: p = 0.5, H_a: p > 0.5$ .
  - $H_0: p > 0.5, H_a: p = 0.5$ .
  - $H_0: p = 0.5, H_a: p \neq 0.5$ .
  - $H_0: p = 0.5, H_a: p < 0.5$ .
  - $H_0: p \neq 0.5, H_a: p > 0.5$ .
- In a test of  $H_0: \mu = 100$  against  $H_a: \mu \neq 100$ , a sample of size 10 produces a sample mean of 103 and a  $P$ -value of 0.08. Which of the following is true at the 0.05 level of significance?
  - There is sufficient evidence to conclude that  $\mu \neq 100$ .
  - There is sufficient evidence to conclude that  $\mu = 100$ .
  - There is insufficient evidence to conclude that  $\mu = 100$ .
  - There is insufficient evidence to conclude that  $\mu \neq 100$ .
  - There is sufficient evidence to conclude that  $\mu > 103$ .
- Which of the following is *not* a required condition for performing a  $t$ -test about an unknown population mean  $\mu$ ?
  - The data can be viewed as a simple random sample from the population of interest.
  - The population standard deviation  $\sigma$  is known.
  - The population distribution is Normal or the sample size is large (say  $n > 30$ ).
  - The data represent  $n$  independent observations.
  - All four of the above are required conditions.

5. An appropriate 95% confidence interval for  $\mu$  has been calculated as  $(-0.73, 1.92)$  based on  $n = 15$  observations from a population with a Normal distribution. If we wish to use this confidence interval to test the hypothesis  $H_0: \mu = 0$  against  $H_a: \mu \neq 0$ , which of the following is a legitimate conclusion?
- Reject  $H_0$  at the  $\alpha = 0.05$  level of significance.
  - Fail to reject  $H_0$  at the  $\alpha = 0.05$  level of significance.
  - Reject  $H_0$  at the  $\alpha = 0.10$  level of significance.
  - Fail to reject  $H_0$  at the  $\alpha = 0.10$  level of significance.
  - We cannot perform the required test since we do not know the value of the test statistic.
6. Which of the following increases the power of a significance test?
- Using a two-tailed test instead of a one-tailed test.
  - Decreasing the size of your sample.
  - Finding a way to increase the population standard deviation  $\sigma$ .
  - Increasing the significance level  $\alpha$ .
  - Decrease the effect size.
7. The infamous psychologist, Dr. Visegrips, claims that his secret sleep tapes cause people to become better at basic algebra “All you have to do,” the doctor explains, “is listen to my tapes while you sleep at night, and you’ll be better at algebra in two months” A math teacher at a local high school has expressed interest but demands evidence. Five people are randomly selected from students at the school. They take an algebra skills test, listen to Dr. Visegrips’ tape for two months while they sleep, and then take a second test. The test scores are as follows:

Person	Test scores				
	A	B	C	D	E
Pre-test	68	69	74	71	65
Post-test	70	68	75	72	68

- Which of the following conditions must be met in order to use a  $t$ -procedure on these paired data?
- The distribution of both pre-test scores and post-test scores must be approximately Normal.
  - The distribution of pre-test scores and the distribution of differences (after – before) must be approximately Normal.
  - Only the distribution of pre-test scores must be approximately Normal.
  - Only the distribution of differences (after – before) must be approximately Normal.
  - All three distributions—before, after, and the difference—must be approximately Normal.
8. Bags of a certain brand of tortilla chips claim to have a net weight of 14 ounces. Net weights actually vary slightly from bag to bag and are Normally distributed with mean  $\mu$ . A representative of a consumer advocacy group wishes to see if there is any evidence that the mean net weight is less than advertised and so intends to test the hypotheses
- $$H_0: \mu = 14 \qquad H_a: \mu < 14$$
- A Type I error in this situation would mean
- concluding that the bags are being underfilled when they actually aren’t.
  - concluding that the bags are being underfilled when they actually are.
  - concluding that the bags are not being underfilled when they actually are.
  - concluding that the bags are not being underfilled when they actually aren’t.
  - none of these

9. A medical researcher is working on a new treatment for a certain type of cancer. After diagnosis, the average survival time on the standard treatment is two years. In an early trial, she tries the new treatment on five subjects and finds that they have an average survival time of four years after diagnosis. Although the survival time has doubled, the results of a  $t$ -test for mean survival time are not statistically significant even at the 0.10 significance level. Which of the following is the best course of action for the researcher?
- (a) Since the test was not statistically significant, she should abandon study of this treatment and move on to more promising ones.
  - (b) She should reexamine her computations—it is likely that she made an error.
  - (c) She should increase the significance level of her test so that she rejects the null hypothesis, since the treatment clearly has a positive impact.
  - (d) She should use a  $z$ -test instead of a  $t$ -test.
  - (e) She should expand her research program to include more subjects—this was a very small sample.
10. You are testing the hypothesis that a new method for freezing green beans preserves more vitamin C in the beans than the conventional freezing method. Beans frozen by the conventional methods are known to have a mean Vitamin C level of 12 mg per serving, so you are testing  $H_0 : \mu = 12$  versus  $H_a : \mu > 12$ , where  $\mu$  = the mean amount of vitamin C (in mg per serving) in beans frozen using the new method. You calculate that the power of the test against the alternative  $H_a : \mu = 13.5$  is 0.75. Which of the following is the best interpretation of this value?
- (a) The complement of the probability of making a Type I error.
  - (b) The probability of concluding that the true mean is 12 mg/serving when it is actually 13.5 mg/serving.
  - (c) The probability of concluding that the true mean is higher than 12 mg/serving when it is actually 12 mg/serving.
  - (d) The probability of concluding that the true mean is 13.5 mg/serving when it actually 12 mg/serving.
  - (e) The probability of concluding that the true mean is higher than 12 mg/serving when it is actually 13.5 mg/serving.

## Part 2: Free Response

*Show all your work. Indicate clearly the methods you use, because you will be graded on the correctness of your methods as well as on the accuracy and completeness of your results and explanations.*

11. Economists often track employment trends by measuring the proportion of people who are “underemployed,” meaning they are either unemployed or would like to work full time but are only working part-time. In the summer of 2013, 17.6% of Americans were “underemployed.” The mayor of Thicksburg wants to show the voters that the situation is not as bad in his town as it is in the rest of the country. His staff takes a simple random sample of 300 Thicksburg residents and finds that 45 of them are underemployed.

(a) Do the data give convincing evidence that the proportion of underemployed in Thicksburg is lower than elsewhere in the country? Support your answer with a significance test.

(b) Interpret the  $P$ -value from your test in the context of the problem.

12. When the manufacturing process is working properly, NeverReady batteries have lifetimes that follow a slightly right-skewed distribution with  $\mu = 7$  hours. A quality control supervisor selects a simple random sample of  $n$  batteries every hour and measures the lifetime of each. If she is convinced that the mean lifetime of all batteries produced that hour is less than 7 hours at the 5% significance level, then all those batteries are discarded.

(a) Define the parameter of interest and state appropriate hypotheses for the quality control supervisor to test.

(b) Since testing the lifetime of a battery requires draining the battery completely, the supervisor wants to sample as few batteries as possible from each hour's production. She is considering a sample size of  $n = 4$ . Explain why this sample size may lead to problems in carrying out the significance test from (a).

(c) Describe a Type I and a Type II error in this situation and the consequences of each.

(d) The quality control officer is considering changing the significance level of the test to 1%. Discuss the impact this might have on error probabilities and the power of the test, and describe the practical consequences of this change.