# 9.2
## Tests About a Population Proportion

- ✓ STATE and CHECK the Random, 10%, and Large Counts conditions for performing a significance test about a population proportion.
- ✓ PERFORM a significance test about a population proportion.
- ✓ INTERPRET the power of a test and DESCRIBE what factors affect the power of a test.
- ✓ DESCRIBE the relationship among the probability of a Type I error (significance level), the probability of a Type II error, and the power of a test.

## Carrying Out a Significance Test

Recall our basketball player who claimed to be an 80% free-throw shooter. In an SRS of 50 free-throws, he made 32. His sample proportion of made shots, 32/50 = 0.64, is much lower than what he claimed.

Does it provide *convincing* evidence against his claim?

### Conditions For Performing A Significance Test About A Proportion

- **Random**: The data come from a well-designed random sample or randomized experiment.
  - **10%**: When sampling without replacement, check that n ≤ (1/10)*N*. Independent, we can use $\sigma = \sqrt{\frac{p(1-p)}{n}}$
- **Large Counts**: Both $np_0$ and $n(1 - p_0)$ are at least 10. Normal

Random-Stated

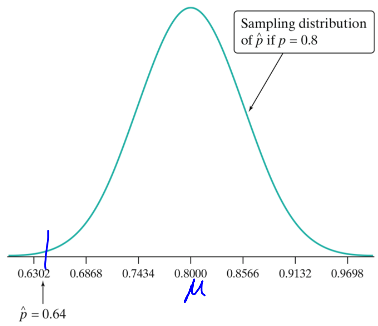
$50 \le \frac{1}{10}(\text{All H.S. F.T. shots})$ yes

~~50(.8)~~

$50(.2) \ge 10$

$10 \ge 10$ yes

$$\mu_{\hat{p}} = p = 0.80 \text{ and standard deviation } \sigma_{\hat{p}} = \sqrt{\frac{p(1-p)}{n}} = \sqrt{\frac{(0.8)(0.2)}{50}} = 0.0566$$



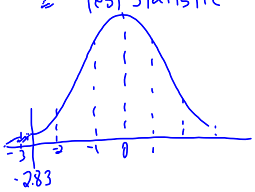

A significance test uses sample data to measure the strength of evidence against $H_0$. Here are some principles that apply to most tests:

- The test compares a statistic calculated from sample data with the value of the parameter stated by the null hypothesis.
- Values of the statistic far from the null parameter value in the direction specified by the alternative hypothesis give evidence against $H_0$.

A **test statistic** measures how far a sample statistic diverges from what we would expect if the null hypothesis $H_0$ were true, in standardized units. That is,

$$\text{test statistic} = \frac{\text{statistic - parameter}}{\text{standard deviation of statistic}} = \frac{\hat{p} - p}{\sqrt{\frac{p(1-p)}{n}}}$$

Z - test statistic




$H_0$: p = 0.80 $\alpha = .05$
$H_a$: p < 0.80

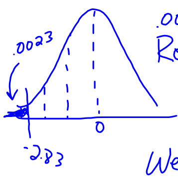
P-value method: 1-PropZTest OR

Standardizing, we get

$$\text{test statistic} = \frac{\text{statistic - parameter}}{\text{standard deviation of statistic}}$$

$$z = \frac{0.64 - 0.80}{0.0566} = -2.83$$

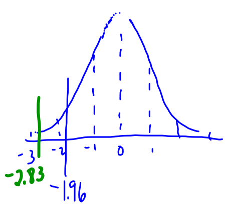
Using Table A, we find that the P-value is P(z ≤ - 2.83) = 0.0023.




.0023 < .05
Reject Ho

**Critical Value Method:**

Critical value Z= $z^* = -1.96$




Reject Ho

We have strong evidence that he shots less than .80 of his Free Throws.

## The One-Sample $z$-Test for a Proportion

**State:** What *hypotheses* do you want to test, and at what *significance level*? Define any *parameters* you use.
**Plan:** Choose the appropriate inference *method*. Check *conditions*.
**Do:** If the conditions are met, perform *calculations*.

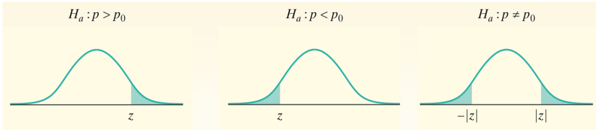
- Compute the **test statistic**.
- Find the ***P*-value**.

**Conclude:** Make a *decision* about the hypotheses in the context of the problem.

## The One-Sample $z$-Test for a Proportion

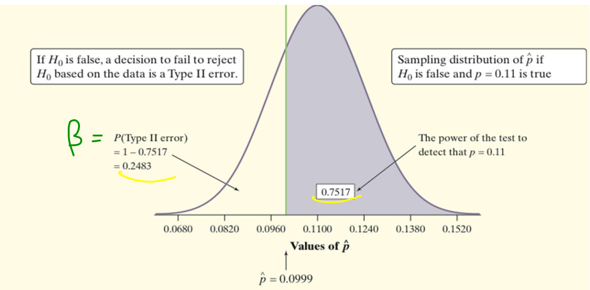
compute the $z$ statistic

$$z = \frac{\hat{p} - p}{\sqrt{\dfrac{p_0(1 - p_0)}{n}}}$$




## Type II Error and the Power of a Test

The potato-chip producer wonders whether the significance test of $H_0: p = 0.08$ versus $H_a: p > 0.08$ based on a random sample of 500 potatoes has enough power to detect a shipment with, say, 11% blemished potatoes. In this case, a particular Type II error is to fail to reject $H_0: p = 0.08$ when $p = 0.11$.




The significance level of a test is the probability of reaching the *wrong* conclusion when the null hypothesis is true. Type I

The power of a test to detect a specific alternative is the probability of reaching the *right* conclusion when that alternative is true. Don't commit Type 2

We can just as easily describe the test by giving the probability of making a Type II error (sometimes called $\beta$).

$\beta = 1 - \text{power}$

34. $200(.99) \geq 10$
$200(.01) \geq 10$
$2 \ngeq 10$ Fails

42. $x = 36$ $n = 50$

$H_0: p = .5$
$H_a: p > .5$

Conditions:
Random: Stated
10% Rule (Indp): $50 \leq \frac{1}{10}$(All Coffee Drinkers)
Large Counts (Norm): $50(.5) \geq 10$
$25 \geq 10$
Met

1-PropZtest

$z = 3.11$ p-value $= .0009$

$.0009 < .05$
Reject $H_0$

We have strong evidence that the proportion of C. Drinkers who prefer Fresh B. is greater than .5