8.2

Estimating a Population Proportion

- STATE and CHECK the Random, 10%, and Large Counts conditions for constructing a confidence interval for a population proportion.
- DETERMINE critical values for calculating a C % confidence interval for a population proportion using a table or technology.
- CONSTRUCT and INTERPRET a confidence interval for a population proportion.
- DETERMINE the sample size required to obtain a C % confidence interval for a population proportion with a specified margin of error.

Suppose one SRS of beads resulted in 107 red beads and 144 beads of another color. The point estimate for the unknown proportion p of red beads in the population would be

$$\hat{p} = \frac{107}{251} = 0.426$$

How can we use this information to find a confidence interval for p?

Before constructing a confidence interval for p, you should check some important conditions

Conditions for Constructing a Confidence Interval About a Proportion

- **Random:** The data come from a well-designed random sample or randomized experiment.
  - **10%:** When sampling without replacement, check that $n \le \frac{1}{10}N$ Independent } S.D. Formula ok to use.
- **Large Counts:** Both $n\hat{p}$ and $n(1-\hat{p})$ are at least 10.

Normal

One-Sample z Interval for a Population Proportion

Once we find the critical value z^*, our confidence interval for the population proportion p is

statistic ± (critical value) · (standard deviation of statistic)

$$z^* = z_{\alpha/2}$$

One-Sample z Interval for a Population Proportion

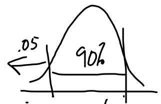
When the conditions are met, a C% confidence interval for the unknown proportion p is

$$\hat{p} \pm z^* \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

where z^* is the critical value for the standard Normal curve with C% of its area between $-z^*$ and z^*.

Suppose you took an SRS of beads from the container and got 107 red beads and 144 white beads. Calculate and interpret a 90% confidence interval for the proportion of red beads in the container. Your teacher claims 50% of the beads are red. Use your interval to comment on this claim.

$$\hat{p} \pm \overbrace{z^* \sqrt{\frac{p(1-p)}{n}}}^{ME}$$



inv Norm(.05,0,1)

$$.426 \pm 1.645 \sqrt{\frac{.426(1-.426)}{251}}$$

$$(.374, .477)$$

use STAT TESTS 1-PropZint

We have evidence that the teacher has overestimated the proportion of Red beads because the interval doesn't contain .5

$$\hat{p} \pm z^* \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} = 0.426 \pm 1.645 \sqrt{\frac{(0.426)(1-0.426)}{251}}$$

$$= 0.426 \pm 0.051$$

$$= (0.375, 0.477)$$

The Four Step Process

We can use the familiar four-step process whenever a problem asks us to construct and interpret a confidence interval.

Confidence Intervals: A Four-Step Process

State: What *parameter* do you want to estimate, and at what *confidence level*?

Plan: Identify the appropriate inference *method*. *Check conditions.*

Do: If the conditions are met, *perform calculations.*

Conclude: *Interpret* your interval in the context of the problem.

Choosing the Sample Size

In planning a study, we may want to choose a sample size that allows us to estimate a population proportion within a given margin of error.

Calculating a Confidence Interval

To determine the sample size n that will yield a level C confidence interval for a population proportion p with a maximum margin of error ME, solve the following inequality for n:

$$z^* \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} \le ME$$

where $\hat{p}$ is a guessed value for the sample proportion. The margin of error will always be less than or equal to ME if you take the guess $\hat{p}$ to be 0.5.

$$n = \frac{(z^*)^2 \, p(1-p)}{(ME)^2}$$

Example: Determining sample size

A company has received complaints about its customer service. The managers intend to hire a consultant to carry out a survey of customers. Before contacting the consultant, the company president wants some idea of the sample size that she will be required to pay for. One critical question is the degree of satisfaction with the company's customer service, measured on a five-point scale.

The president wants to estimate the proportion p of customers who are satisfied (that is, who choose either "satisfied" or "very satisfied," the two highest levels on the five-point scale). $2/5$

She decides that she wants the estimate to be within $\underbrace{3\%\ (0.03)}_{ME}$ at a 95% confidence level.

How large a sample is needed?

$$n = \frac{(1.96)^2(.4)(.6)}{(.03)^2} = 1024.42$$

$$n = 1025$$

If p or a previos $\hat{p}$ are not given then use .5

$$n = \frac{(1.96)^2(.5)(.5)}{(.03)^2} = 1067.11$$

$$1068$$